A numerical analysis of Japan's fiscal sustainability in a simple OLG model

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Abstract

We investigate (i) the size of primary balance that can be sustained and (ii) the change in public debt and physical capital under transition dynamics in the Japanese economy. For our investigation, we construct a simple overlapping generations (OLG) model. We find that a large primary surplus, 13.8–18.7% of GDP, is needed to prevent the public debt-to-GDP ratio from diverging infinitely. We also show that even if the large primary surplus can be maintained, the Japanese economy faces a sharp reduction in physical capital from 191% to 70.0% of GDP on the transition path.

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I. Introduction

Japan's public debt has been increasing for the past few decades. The financial net liabilities of Japan's general government increased from 22.8% of GDP in 1995 to 135.9% in 2012.[1] Under such a circumstance, it is an important policy issue to investigate what fiscal policies prevent the public debt-to-GDP ratio from diverging and what happens on the transition path under sustainable fiscal policies.

We investigate two questions. First, what size of a primary balance can be sustained in Japan? Second, if the government minimally improves the primary balance, how much do public debt and physical capital change under transition dynamics in the Japanese economy. For our investigation, we construct a simple overlapping generations model and numerically investigate the questions. Although there are a number of previous studies that quantitatively investigate the dynamics of Japan's public debt (Ihori *et al.*, 2006; Ishikawa *et al.*, 2012; Hansen and İmrohoroğlu, 2012), these assume that the public debt-to-GDP ratio converges to a finite level that is given exogenously.[2]

[1] The data source is OECD Outlook 93 database.

[2] There are a few exceptions. İmrohoroğlu and Sudo (2011) show that unprecedentedly high growth rates are necessary to reduce the public debt-to-GDP ratio. However, they take fiscal variables and payments of interest as exogenous variables and do not take into account general equilibrium effects, unlike in this study. Arai and Ueda (2013) investigate the maximum size of the primary deficit-to-GDP ratio that prevents the debt-to-GDP ratio from diverging with consideration for general equilibrium effects. However, they derive only a necessary condition for fiscal sustainability because their analysis focuses on the steady state. This study complements the literature on quantitative investigation of sustainable fiscal policies.

In their studies, however, a *set* of sustainable fiscal policies for Japan is yet to be ascertained, and we intend to do so in this paper.

II. Model

Our model is based on that of Chalk (2000). We consider an overlapping generations model in which households live for two periods (young and old). Uncertainty and intra-generational heterogeneity are assumed away. The population of each generation born in period *t*, N_t, grows at a rate of (1+*n*). The young supply their labor inelastically, receive labor wages and consume and/or save their disposable income. The old deplete and consume their savings and asset incomes. Households born at period *t* have a utility function given by $c_t^{1-\sigma}/(1-\sigma)+\beta d_{t+1}^{1-\sigma}/(1-\sigma)$, where $\beta \in (0,1)$ is the subjective discount rate, σ is the inverse of the intertemporal elasticity of substitution, and c_t and d_{t+1} denote consumption when young and old, respectively. Households born at period *t* face an intertemporal budget constraint as follows:

$$(1+\tau^c)c_t+\frac{(1+\tau^c)d_{t+1}}{1+(1-\tau^k)(R_{t+1}-1)} = (1-\tau^l-\tau^s)w_t+\frac{\xi w_t}{1+(1-\tau^k)(R_{t+1}-1)} \quad (1)$$

where R_{t+1} is an interest rate and w_t is a wage rate. τ^l, τ^k and τ^c are labor income, capital income and consumption tax rates, respectively, which are assumed to be exogenous and time-invariant. τ^s is a contribution rate of social security pensions and ξ is a replacement rate. Each household maximizes its lifetime utility subject to the intertemporal budget constraint. Solving the utility-maximization problem, the amount of saving, s_t, is derived as

$$s_t = \frac{(1-\tau^l-\tau^s)w_t + \xi w_t/[(1-\tau^k)(R_{t+1}-1)+1]}{1+\beta^{-1/\sigma}[(1-\tau^k)(R_{t+1}-1)+1]^{1-1/\sigma}} \qquad (2)$$

A representative firm produces final goods using labor and physical capital in a perfectly competitive market. A production function is Cobb-Douglas, $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$, where Y_t is output, A_t is total factor productivity (TFP, henceforth) that grows at rate $(1+\lambda)$, K_t is aggregate physical capital, L_t is aggregate labor supply and $\alpha \in (0,1)$ is an exogenous parameter. The profit-maximization conditions are $r_t = A_t \alpha K_t^{\alpha-1} L_t^{1-\alpha}$ and $w_t = A_t(1-\alpha)K_t^{\alpha} L_t^{1-\alpha}$. A government maintains the ratio of government expenditure, G_t, to GDP at a constant level in all periods. That is, for all t, $G_t/Y_t = G$, where G is exogenously given. Government expenditure is assumed to be wasteful. The government follows flow budget constraints in every period, $B_{t+1} = R_t B_t + G_t + \xi w_{t-1} N_{t-1} - T_t$, where B_t is an amount of public debt at the beginning of period t. T_t is total tax revenues:

$$T_t = \tau^l w_t N_t + \tau^s w_t N_t + \tau^k (R_t - 1) s_{t-1} N_t + \tau^c (c_t N_t + d_t N_{t-1}) \qquad (3)$$

We consider a closed economy, and thus, all markets must be clear in all periods. The labor and capital market clearing conditions are $L_t = N_t$ and $K_{t+1} = s_t N_t - B_{t+1}$, respectively. We also assume a no-arbitrage condition between private investment and public debt in a competitive equilibrium. Then, for all t, $(1-\tau^k)(R_t - 1) = (1-\tau^k)(r_t - \delta)$.

Lastly, we define a competitive equilibrium and a steady state. We define a *competitive equilibrium* as a sequence of predetermined variables and a price system that solves the household and firm's optimization problems and satisfies the market clearing and non-arbitrage conditions for all t, given initial amounts of public debt and physical capital, B_0 and K_0, and policy variables, G, τ^l, τ^k, τ^c, τ^s and ξ. We also define a *steady state* as the competitive equilibrium in which the predetermined variables grow at the same rate of $(1+\lambda)^{1/(1-\alpha)}\,(1+n)$ for all t.

III. Numerical Calculation

Calibration

We assume that the time period corresponds to 40 years in our model. The subjective discount rate is $\beta = (0.9676)^{40} = 0.2678$, which is based on the calibration of Arai and Ueda (2013). The intertemporal elasticity of substitution is set to 0.8, and thus, $\sigma = 1/0.8 = 1.25$.[3] The share of capital income is calculated as $\alpha = 0.3647$. The annualized depreciation rate of physical capital is set to 0.0888.[4] The population growth and TFP growth rates are exogenously given as $n = 0.0$ and $\lambda = 0.2877$.[5] We adopt the tax rates and the contribution rate of social security corresponding to those in the current Japanese economy, which gives $\tau^l = 0.16$, $\tau^s = 0.10$, $\tau^k = 0.30$ and

[3] The values of preference parameters used here are very similar to those adopted in simulations of Japanese economy (e.g., Chen *et al.* 2007; Braun *et al.* 2009).

[4] We use the average values of capital income share, fixed capital depreciation, and physical capital in 2000–2009 to calculate α and δ. These values are obtained from the Japan's SNA statistic account. The stock of physical capital is defined as the sum of corporate capital, non-corporate capital, and (corporate and non-corporate) inventories. It also should be noted that $\delta = 1 - (1 - 0.0888)^{40} = 0.9758$.

[5] We target the yearly growth rate as 1.0%. Using $n = 0$, we obtain

$$\begin{aligned}\lambda &= [(1 + \text{the target yearly growth rate})^{1-\alpha}]^{40} - 1 \\ &= (1+0.01)^{(1-0.3647)40} - 1 = 0.2877. \end{aligned} \tag{4}$$

$\tau^c = 0.10$.[6] The replacement rate is set to $\xi = 0.3456$, which is calibrated so as to match to the Japan's actual replacement rate in 2009 obtained from the Japan's SNA statistic account.

We set initial capital-to-GDP and public debt-to-GDP ratio to $K_0/Y_0 = 0.0682$ and $B_0/Y_0 = 0.0394$, respectively, by using actual values of the capital-to-GDP and public debt-to-GDP ratios in 2009, and averages of economic growth and interest rate in 2000–2009.

Numerical calculation

In this section, we derive the maximum size of sustainable government spending, using the parameters calibrated in the previous subsection. We show that if government spending is kept to below 7.98% of GDP, the Japanese economy can prevent the public debt-to-GDP ratio from diverging infinitely in our model. When the government maintains its spending per GDP at 7.98%, the dynamics of physical capital and public debt per GDP are as illustrated in Fig. 1. Fig. 1 implies that Japan's government has to improve the fiscal balance drastically to achieve fiscal sustainability. Fig. 1 illustrates that the primary surplus must be kept over 13.8–18.7% of GDP for quite a few periods (corresponding to around 160–200 years).

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[6] The labor income tax and contribution rates are calculated by using the Japan's SNA statistic account (the household's secondary distribution of income account and the social security contributions). The capital income tax rate is set as in Arai and Ueda (2013). The Japan's consumption tax rate is scheduled to rise to 10% in 2015.

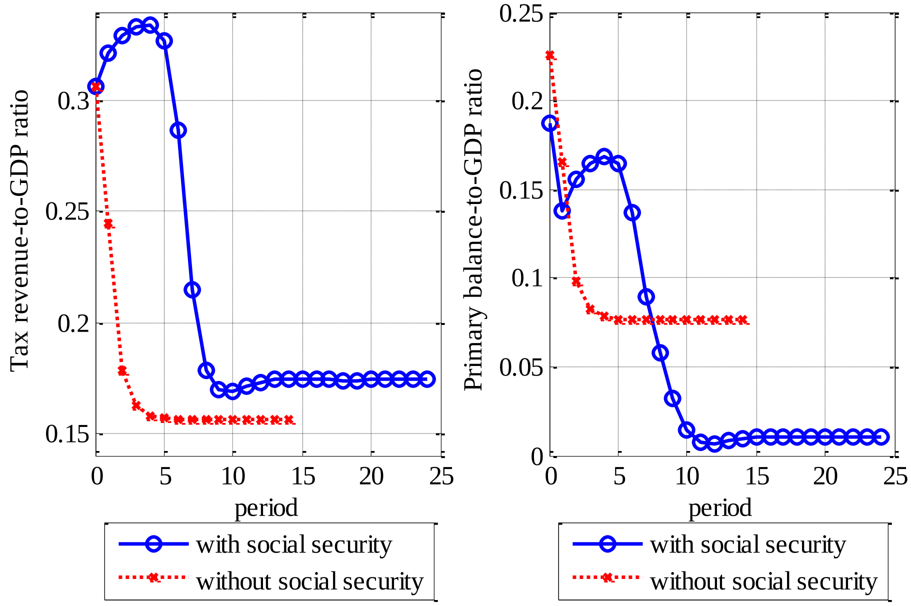

Fig. 1: The path of tax revenue-to-GDP and primary balance-to-GDP ratios under the maximum sustainable size of government spending-to-GDP ratio ($G = 0.0798$)

Notes: In both figures, the blue solid (red dotted) line corresponds to the case where the social security system is (is not) introduced.

Furthermore, even if the government can maintain the primary surplus-to-GDP ratio at a sufficiently higher level, the Japanese economy will face a decline in the amount of physical capital for some periods, which is illustrated in Fig. 2. From $t = 0$ to $t = 5$, the (annualized) physical capital-to-GDP ratio decreases from 191% to 70.0%.

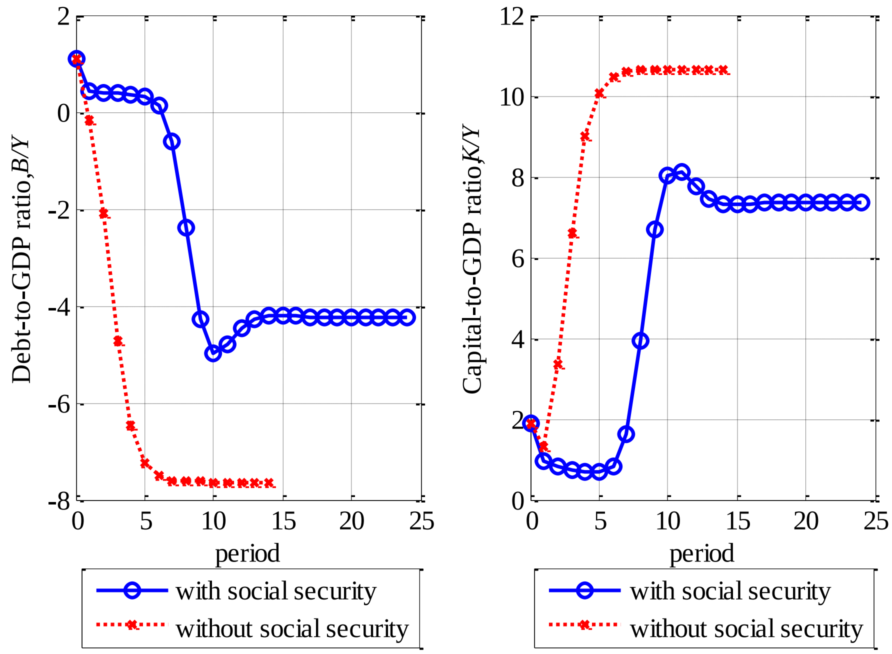

Fig. 2: The path of public debt-to-GDP and physical capital-to-GDP ratios under the maximum sustainable size of government spending-to-GDP ratio ($G = 0.0798$)

Notes: The blue solid (red dotted) line corresponds to the case where the social security system is (is not) introduced.

Why does the physical capital-to-GDP ratio substantially fall? This is because of two important factors. One is the large amount of the initial public debt per GDP, which generates a large crowd-out effect on private investment. The other is the social security system. Fig. 2 also illustrates that if the social security system were shut down ($\xi = \tau^s = 0$), it could mitigate the fall in the capital-to-GDP ratio and the recovery of the capital-to-GDP ratio is moved forward in some decades. The intuition underlying this is as follows: even if government expenditure is sufficiently small, the public debt per GDP does not decrease because the government must spend the revenue for public pensions, which leads to a decrease in the physical capital per GDP for the first few periods. After the capital per GDP lowers to some extent, the amount of public debt begins to decrease as the payment of public pensions does. As the public debt per GDP decreases, the private investment and physical capital per GDP grow.

Lastly, we would like to emphasize that the key insight is the dynamics of fiscal and macroeconomic variables *in the first several decades (from* $t = 0$ *to around* $t = 5$ *in the model)*. Although the capital-to-GDP and public debt-to-GDP ratios seem to eventually reach unrealistic levels in our analysis, we consider that they do not have important implications.[7]

IV. Conclusion

We obtain two quantitative results on Japan's fiscal sustainability. First, given realistic tax and social security systems, the government has to maintain the primary surplus at 13.8–18.7% of GDP in the next few decades. Second, even if the government can maintain the primary surplus at the sustainable level, the Japanese economy faces a decrease in physical capital per GDP from 191% to 70.0% of GDP on the sustainable dynamic path.

[7] After Japan's government reduces the public debt sufficiently in the distant future, the government seems to increase public spending and/or cut tax rates. This fiscal relaxation leads to a higher public debt-to-GDP ratio and a lower capital-to-GDP ratio than those obtained in our calculation.

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