Productive Government Expenditure and Fiscal Sustainability

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Abstract

We consider an overlapping generations model in which public spending directly contributes to an increase in productivity, as in the model of Barro (1990), and in which the government conforms to the constant public-spending/GDP and debt-issuance/public-spending ratio rules. We numerically analyze the effects of a change of the public-spending/GDP ratio on fiscal sustainability, growth rate, and welfare. First, if the public-spending/GDP ratio is small, an increase in the ratio makes public debt sustainable. Second, if the public-spending/GDP ratio is small, then an increase in the public-spending/GDP ratio is Pareto improving.

Keywords: fiscal sustainability, welfare, productive government spending, public debt, endogenous growth

JEL classification: E62, H53, H63

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1 Introduction

Recently, public debt has grown or stayed at high level in most developed countries. For example, the gross public-debt/GDP ratio of Greece reached 147.3% in 2010[1]. and Greece now faces a debt crisis. The gross public-debt per GDP of Japan was 199.7% in 2010. Under these circumstances, the aim of achieving fiscal sustainability has stimulated many countries to make an effort to avoid accumulating excessive public debt. EU countries are trying to meet the fiscal constraints of the Maastricht treaty convergence criteria, in which the governments must avoid a budget deficit of more than 3 % of GDP and their debt/GDP ratio must not exceed 60%.

In many countries, governments are trying to decrease their public spending or to attain higher economic growth in order to prevent their public debt from growing excessively while simultaneously trying to observe fiscal disciplines such as the Maastricht treaty convergence criteria. Some EU countries, for example Greece, Portugal and Spain, have announced that their government expenditure has declined in the last few years in order to meet the Maastricht treaty convergence criteria. In 2006, the Japanese prime minister, Junichiro Koizumi, unveiled a plan whereby the new-issuance of public debt is capped at 30,000 billion yen.

Meanwhile, some people consider that achieving higher economic growth is a more sustainable fiscal policy. Domar (1944) stated that if economic growth is sufficiently large then present public debt can be sustainable. This paper is often cited in discussions of economic policy.

In response to this situation, we ask the following questions. *Can governments always prevent their public debt from growing by decreasing their public spending? Does growth-maximizing fiscal policy always make public debt sustainable? Does the policy which makes present public debt more sustainable improve the welfare of households?*

We consider an economy where public spending directly contributes to an increase in pro-

[1]The data of Greece and Japan are obtained from *OECD Outlook database*, which is available on the OECD website

ductivity and investigate what types of policy change lead to a more sustainable fiscal policy and how a change in the fiscal policy affects households' welfare. *A fiscal policy is more sustainable than another policy* if one fiscal policy gives the higher upper limit of initial public-debt/capital ratio for which the public-debt/capital ratio converges to some finite level than the upper limit given by another policy. The concept of a more sustainable fiscal policy is used to investigate whether larger amounts of public debt can continue to be repaid and rolled over by a change of a fiscal policy rule. Intuitively, we can interpret a more sustainable fiscal policy (than another policy) to mean that, starting from some similar initial public-debt/capital ratio, the public-debt/capital ratio can converge to some finite level under one fiscal policy but the ratio diverges under another policy. In this paper, we focus on the case where the government employs a constant public-spending/GDP ratio and constant debt-issuance/public-spending ratio policies.

We numerically analyze the effect of productive government expenditure on fiscal sustainability. We obtain the following two main results in this paper: (i) There exists an inverse-U shaped relationship between the size of public-spending and the level of sustainable public debt, and the growth-maximizing public-spending level does not always make public debt sustainable. (ii) When the public-spending/GDP ratio is small, an increase in the ratio is Pareto improving.

An increase in the public-spending/GDP ratio has three effects. First, an increase in the public-spending/GDP ratio improves productivity and promotes the accumulation of private capital and economic growth. This is a positive effect for fiscal sustainability. Second, it raises the income tax rate because the public-spending/GDP ratio rises and part of it must be financed by income tax. Third, it results in higher interest rates and accumulates more public debt. This negatively impacts fiscal sustainability due to increasing in the payment of interest, and crowding-out of private capital.

It is worth noting that the first and second effects are obtained in the model of Barro (1990), and the third effect is an additional effect investigated in this paper. Because of the third effect, we not only obtain the inverse-U shaped relationship between the public-spending/GDP ratio and maximum sustainable initial debt level, but also find that growth-maximizing public-spending/GDP ratio is not equal to the public-spending/GDP ratio which gives maximum sustainable initial debt level.

The second result is that when an increasing public-spending/GDP ratio makes the largest sustainable public debt higher, the policy change is Pareto improving. Increasing the public-spending/GDP ratio does not necessarily make all generations' welfare higher, especially for those on the transition path.

The welfare analysis is not trivial. The first result implies that the fiscal policies that give higher output growth do not correspond to more sustainable fiscal policies. The analysis of the relationship between fiscal sustainability and the welfare of each generation does not follow from the result of Barro (1990). Furthermore, an increase in the public-spending/GDP ratio leads to higher interest rates, and it is uncertain whether the public-debt/capital ratio rises or falls at the (stable) steady state even if the increase leads to a more sustainable fiscal policy. A rise in the public-debt/capital ratio leads to larger tax-payments and retarded output growth through the third effect in our model. If the third effect is larger than the other two effects, the welfare of some households may worsen.

Many past studies have investigated the relationship between productive public spending and economic growth. Barro (1990) introduces public flow spending, which is directly used to raise productivity and affects the growth rate. In contrast, Futagami et al. (1993) consider the case where public capital investment raises productivity, and they analyze the dynamics of public-capital/private-capital ratio and the policy effects on economic growth. These studies focus on the effect of fiscal policy on economic growth and they do not discuss fiscal sustainability.

Meanwhile, the relationship between fiscal policy rules and economic performance has been investigated by several authors. For example, Agénor and Yilmaz (2011) investigate how a policy change affects the transition path of public debt and economic growth rate, and the rate of convergence to a new steady state under various fiscal policy rules. Moreover, they point out that high initial public debt leads to unstable dynamics in their model where the government spends for productive expenditure and conforms to some fiscal policy rules. However, they do not discuss how large initial amounts of public debt leads to stable dynamics. In contrast, we focus on the initial size of public-debt/capital ratio which leads to stable dynamics under various fiscal policies.

In recent years, some studies have analyzed fiscal sustainability under the assumption that the government issues public debt according to a given fiscal policy rule. Chalk (2000) and Rankin and Roffia (2003) examine the condition for fiscal sustainability in exogenous-growth overlapping generations (OLG) models given fiscal policy rules. Bräuninger (2005) also investigates the conditions for fiscal sustainability in an endogenous-growth OLG model in which the government conforms to constant public-spending/GDP ratio and constant public-debt issuance/GDP ratio rules. However, the studies of fiscal sustainability introduced above have not considered the effect of government expenditure on productivity.

A limited number of studies have analyzed the welfare effect under the constant public-spending/GDP ratio and public debt issuance/public-spending ratio rules. Saint-Paul (1992) and Futagami and Shibata (1998) show that changes in public spending can never Pareto improve in an endogenous growth OLG model[2]. They take into account the accumulation of public debt and analyze the welfare of all generations, but they also assume that the expenditure is unproductive. Mourmouras and Lee (1999) introduce the productive government expenditure following the

[2]Note that they assume that the government conforms to the different fiscal policy rules (as stated in studies described above), which are constant public-spending/private capital and *lump-sum tax*/private capital ratios.

model of Barro (1990) and show that the growth-maximizing level of spending also maximizes the utility of all households. However, they only consider the balanced-budget case. Tanaka (2003) allows the case where the government can issue public debt, and analyzes the welfare effect, which is close to our study. However, he focuses the analysis on the case when a policy parameter is fixed to only one value. In contrast, we analyze the welfare effect under various policy parameters.

Our research is not the first study to tackle the analysis of the relationship between productive government expenditure and fiscal sustainability. Yakita (2008) shares the motivation for analyzing fiscal sustainability in a growth model with productive government expenditure. Yakita (2008)'s study is based on work by Futagami et al. (1993) and assumes that the government invests in public capital. Meanwhile, our study follows Barro (1990). In our model, we assume that the government flow expenditure instantaneously increases productivity. In addition, Yakita (2008) does not investigate the welfare effect.

The reason we must consider the case of productive government expenditure rather than public investment is that the government actually spends not only on public investment but also on other uses which may be productive; for instance, public peace, defense, establishment and maintenance of the judicial system, and so on. The government expenditure outside of public investment takes a large part of the budget[3]. Furthermore, there are some empirical results showing that expenditure can contribute to economic growth. For example, Aizenman and Glick (2006) indicate that military expenditure positively affects economic growth when external threats are sufficiently large; Barro and Sala-i-Martin (2003) find from empirical analysis that the degree of the rule of law positively affects economic growth.

The results from our model are consistent with some empirical studies on the relationship

[3]For example, in Japan (in fiscal year 2006), the ratio of the public final consumption to GDP was 11.27%, while the ratio of the public gross capital formation to GDP was 4.21%. We calculate the public final consumption by subtracting the final consumption of social security funds from the total public final consumption.

between fiscal policy and economic growth. For example, Kneller et al. (1999) used a panel of 22 OECD countries in 1970 to 1995 and found that productive government spending enhances economic growth. The empirical findings of Kneller et al. (1999) are consistent with the setting of our model. Adam and Bevan (2005) find that the relationship between public-deficit/GDP ratio and economic growth is non-linear in developing countries: when public deficit is high (low), the contraction of the deficit gives large (small) economic growth. This relationship is also obtained in our model.

The rest of this paper is organized as follows. In section 2, we present the framework of our model and analytically investigate policy effects on fiscal sustainability and welfare of households. We numerically analyze the policy effects on fiscal sustainability, economic growth, and welfare in section 3. Finally, section 4 presents the conclusions.

2 Model

We consider an OLG model that consists of individuals, firms, and the government.

2.1 Model Setup

Individuals live for two periods. The size of population of each generation is the same, which is normalized to unity. Individuals who are born at $t = 0, 1, 2, \cdots$ have an identical utility function as below,

$$\ln c_t^y + \beta \ln c_{t+1}^o \tag{1}$$

where c_t^y is their consumption when they are young, c_{t+1}^o is their consumption when they are old, and $\beta \in (0, 1)$ is the subjective discount rate. When individuals are young, they supply their own labor inelastically, receive wages and consume or save their wages. On the other hand, when old, they deplete their savings. There are no bequests. Each household maximizes its own utility subject to an intertemporal budget constraint.

Firms produce goods in a perfectly competitive market. They have identical production technology,

$$y_t = Ak_t^{\alpha}(g_t l_t)^{1-\alpha} \tag{2}$$

where A is the technological parameter (constant), k_t is the stock of physical capital, g_t is the government expenditure per worker, and l_t is unit of worker. This production function is the same as that of Barro (1990). Firms aim to maximize their profit in a perfectly competitive market; thus, we can write the firm's profit maximization problem as

$$\max_{k_t, l_t} \pi_t = y_t - r_t k_t - w_t l_t. \tag{3}$$

The government allocates public expenditures in order to raise the productivity of labor. The government spending is financed by a flat-rate income tax or issuance of public debt.

$$\begin{aligned} b_{t+1} &= (1 + R_t)b_t + g_t - T_t \\ T_t &= \tau_t[r_t s_{t-1} + w_t] \end{aligned} \tag{4}$$

where we denote b_t as the stock of public debt at the initial period t, R_t as the interest rate of public debt, and T_t as total tax revenue, respectively. We assume that the government conforms to the fiscal policy rules as follows:

$$g_t = \xi y_t \tag{5}$$

$$b_{t+1} - b_t = \chi g_t. \tag{6}$$

where $\xi, \chi \in [0, 1]$ are policy parameters. Equation (5) means that the government fixes the public-spending/GDP ratio, and Equation (6) states that public debt is issued at a constant proportion of government expenditure. By these rules, the level of public spending and of issuance of public debt are given and so the income tax rate is endogenously determined. In other words, if public spending and/or public debt increase, the income tax rate rises as the government's budget constraint is satisfied. Equations (5) and (6) conform to the budget deficit rules of the Maastricht Treaty convergence criteria and have been used in many previous studies, such as Bräuninger (2005) and Yakita (2008).

2.2 Competitive Equilibrium and Balanced Growth Path

Next we define competitive equilibrium and only consider the path that attains competitive equilibrium in every period.

Definition 1 (competitive equilibrium)**.** *A set of sequences of the predetermined variables* $\{k_t, b_t\}_{t=0}^{\infty}$ *and a price system* $\{r_t, R_t, w_t\}_{t=0}^{\infty}$ *is a* ***competitive equilibrium*** *if, for any* t*, it satisfies the following conditions:*

1. *Households' utility maximization conditions,*

$$s_t = \frac{\beta}{1+\beta}(1-\tau_t)w_t. \tag{7}$$

2. *Firms' profit maximization conditions,*

$$r_t = \alpha A k_t^{\alpha-1} g_t^{1-\alpha} \tag{8}$$

$$w_t = (1-\alpha) A k_t^{\alpha} g_t^{1-\alpha}. \tag{9}$$

3. *The government's budget constraint,* (4)*, and the fiscal policy rules,* (5) *and* (6).

4. *No-arbitrage condition,*

$$(1-\tau_t)R_t = (1-\tau_t)r_t \;\Leftrightarrow\; R_t = r_t. \tag{10}$$

5. *Capital market clearing condition,*

$$s_t = k_{t+1} + b_{t+1}. \tag{11}$$

6. *Labor market clearing condition,*

$$l_t = 1. \tag{12}$$

Furthermore, we define the balanced growth path steady state as follows.

Definition 2 (Balanced growth path steady state)**.** *A set of sequences of predetermined variables* $\{k_t, b_t\}_{t=0}^{\infty}$ *and a price system* $\{r_t, R_t, w_t\}_{t=0}^{\infty}$ *is the* ***balanced growth path steady state (BGP)*** *if it is a competitive equilibrium and there exists some* γ *such that for any* t*,*

$$\frac{k_{t+1}}{k_t} = \frac{b_{t+1}}{b_t} = \gamma. \tag{13}$$

Here, we define the growth rates of capital and of public debt as $\Phi(\tilde{b}_t)$ and $\Psi(\tilde{b}_t)$, respectively. That is,

$$\Phi(\tilde{b}_t) := k_{t+1}/k_t = \frac{\beta}{1+\beta}\frac{1+\xi\chi-\xi}{1+\alpha\tilde{b}_t}A^{1/\alpha}\xi^{1/\alpha-1}(1-\alpha) - \tilde{b}_t - \chi A^{1/\alpha}\xi^{1/\alpha}, \tag{14}$$

$$\Psi(\tilde{b}_t) := b_{t+1}/b_t = 1 + \chi\tilde{b}_t^{-1}A^{1/\alpha}\xi^{1/\alpha}. \tag{15}$$

By defining $\tilde{b}_t := b_t/k_t$, we can express the dynamic system[4] as

$$\frac{\tilde{b}_{t+1}}{\tilde{b}_t} = \frac{\Psi(\tilde{b}_t)}{\Phi(\tilde{b}_t)} = \frac{1 + \chi\tilde{b}_t^{-1}A^{1/\alpha}\xi^{1/\alpha}}{\frac{\beta}{1+\beta}\frac{1+\xi\chi-\xi}{1+\alpha\tilde{b}_t}A^{1/\alpha}\xi^{1/\alpha-1}(1-\alpha) - \tilde{b}_t - \chi A^{1/\alpha}\xi^{1/\alpha}}. \tag{16}$$

By this definition, the system is in a BGP if and only if $\tilde{b}_t = \tilde{b}$ for all t.

We investigate the conditions for existence of the BGPs and show the following proposition.

Proposition 1. *Given* A *and* ξ*, if* A *and* ξ *satisfy*

$$\frac{\beta}{1+\beta}(1-\xi)A^{1/\alpha}\xi^{1/\alpha-1}(1-\alpha) > 1, \tag{17}$$

then there exists a critical value of χ*, which represents as* χ^**, such that*

(I) *if* $\chi < \chi^*$*, the system has two BGPs,*

[4]We show the derivation of equation (16) in Appendix A.

(ii) *if $\chi = \chi^*$, the system has only one BGP, and*

(iii) *if $\chi > \chi^*$, the system has no BGP.*

Proof. The growth rate of capital, Φ, and public debt, Ψ, satisfy the following properties,

- Φ and Ψ are decreasing and convex in $\tilde{b}_t$.
- $\Phi(0) \in (0, \infty)$ and $\lim_{\tilde{b}\to\infty} \Phi(\tilde{b}) < 0$.
- $\lim_{\tilde{b}\to 0} \Psi(\tilde{b}) = +\infty$ and $\lim_{\tilde{b}\to\infty} \Psi(\tilde{b}) = 1$.

Hence, if the equation (17) is satisfied, the system must have two BGPs when $\chi = 0$. Moreover, we obtain $\partial\Phi/\partial\chi < 0$ and $\partial\Psi/\partial\chi > 0$. This means that when χ rises, the graph of Φ shifts upward and the graph of Ψ shifts downward. Thus, we can show that

- When χ is small, there exists two BGPs.
- When χ is large, there exists no BGP.

Figure 1 illustrates that this proposition holds[5]. □

[Figure 1 about here.]

Figure 1 illustrates whether or not there are BGPs. When χ is small, the graph of Φ shifts upward, the Ψ curve shifts downward, and BGPs do tend to exist. If a higher χ is chosen, Φ moves downward, Ψ moves upward, and the BGPs become nonexistent.

2.3 Stability of BGP

In this subsection, we examine the stability of BGP. We obtain the following proposition.

[5]Strictly speaking, we can prove this with the saddle-node bifurcation theorem. See Devaney (2003)

Proposition 2. *When two BGPs exist, the BGP with a smaller $\tilde{b}$ is stable and the other with larger $\tilde{b}$ is unstable.*

Proof. By definition of Φ and Ψ, we get

$$\frac{\tilde{b}_{t+1}}{\tilde{b}_t}\begin{cases}\geq 1 \\ \\ < 1\end{cases} \iff \begin{cases}\Phi(\tilde{b}_t) \leq \Psi(\tilde{b}_t) \\ \\ \Phi(\tilde{b}_t) > \Psi(\tilde{b}_t).\end{cases} \tag{18}$$

Therefore, Figure 2 shows that the statement holds. □

[Figure 2 about here.]

From Proposition 2, we can show that the public-debt/capital ratio in the unstable BGP is the upper limit of initial public-debt/capital ratio for which the public-debt/capital ratio converges to some finite level. Let the public-debt/capital ratio in a stable BGP and the ratio in an unstable BGP be $\tilde{b}^S$ and $\tilde{b}^U$, respectively. From Figure 2, if $\tilde{b}_0 \leq \tilde{b}^U$, then $\tilde{b}_t$ converges to $\tilde{b}^S$ (or remains $\tilde{b}^U$). On the other hand, if $\tilde{b}_0 > \tilde{b}^U$ then $\tilde{b}_t$ diverges. This proposition implies that not only policy parameters but also initial stock of public debt are important for fiscal sustainability. This result is consistent with that of previous studies.

2.4 Policy Effect on Fiscal Sustainability

In this subsection, we analyze the effects of changes in fiscal policies. More specifically, we will investigate the effects on fiscal sustainability, economic growth, and welfare of households when the fiscal policy parameters, ξ and χ, change.

Before we analyze policy effect on fiscal sustainability, we define two terms on sustainability of fiscal policy rules. First, we define *a sustainable fiscal policy*.

Definition 3 (sustainable fiscal policy)**.** *A fiscal policy (ξ, χ) is sustainable if there exists an initial level of public-debt/capital ratio such that the ratio converges to some finite level in the long-run.*

In our model, a sustainable fiscal policy means that, under the fiscal policy, the public-debt/capital ratio can avoid diverging if initial debt/capital ratio is sufficiently low. As above, we can interpret a sustainable fiscal policy as a necessary condition for the public-debt/capital ratio to converge to some finite level: A debtor country can continue to repay and roll over its debt if a fiscal policy is sustainable *and* the initial public-debt/capital ratio is sufficiently small.

Further, a fiscal policy is sustainable if there exists a BGP in this system because when the system has a BGP, the debt/capital ratio can converge for some initial stocks of debt and capital, which is shown in section 2.3.

Next, we define *a more sustainable fiscal policy than another policy*. This concept is used when we analyze policy effects on fiscal sustainability. Note that the meaning of a more sustainable policy differs from the meaning of sustainable policy. This point is explained later.

Definition 4 (a more sustainable fiscal policy)**.** *A fiscal policy* (ξ, χ) *is more sustainable than another fiscal policy* (ξ', χ') *if the fiscal policy* (ξ, χ) *leads to the wider range of initial public-debt/capital ratio for which the public-debt/capital ratio converges to some finite level than the range lead by another policy* (ξ', χ')*.*

From proposition 1 and 2, we can intuitively interpret a more sustainable fiscal policy (than another policy) to mean that, starting from some similar initial public-debt/capital ratio, the public-debt/capital ratio can converge to some finite level under one policy but the ratio does not converge under another policy. The reason is that the range of initial public-debt/capital ratio for which the public-debt/capital ratio converges can be represented as $[0, \tilde{b}^U]$ in our model[6] , and this means that a more sustainable policy leads to higher $\tilde{b}^U$. In the following part, we investigate whether the new fiscal policy is more sustainable than the original policy when a fiscal policy changes.

[6]We have noted that $\tilde{b}^U$ is the upper limit of initial public-debt/capital ratio for which the public-debt/capital ratio converges. Please see proposition 2 and the following sentences.

Here, we clarify the difference between the meaning of a sustainable fiscal policy and a more sustainable fiscal policy. A sustainable fiscal policy is defined by whether there exists the path for the public-debt/capital ratio to converge. Meanwhile, a more sustainable fiscal policy is defined by how large a set of the paths for the public-debt/capital ratio to converge is: comparing two fiscal policies, the policy which leads to a larger set of the paths for the public-debt/capital ratio is called more sustainable than another policy.

We start the analysis of policy effects on sustainability of fiscal policies. In our model, there are two policy parameters, ξ and χ. Hence, we investigate (i) effects of changing χ on fiscal sustainability given ξ, and (ii) effects of changing ξ given χ.

At first, we analyze policy effects on fiscal sustainability when the government changes χ. By proposition 1 and 2, we can easily show that the larger χ the government chooses, the less sustainable the fiscal policy becomes and the less likely BGPs are to exist. Intuitively, an increase in χ causes the public debt growth rate to rise and private capital to be crowded out, and so that the fiscal policy becomes less sustainable.

In contrast, when the government changes ξ under a constant χ, the policy effect on fiscal sustainability may not be the same as that in the case where the government changes χ. To analyze this effect, we focus on the unstable BGP, in which $\tilde{b}_t = \tilde{b}^U$. In the BGP, we have

$$\Phi(\tilde{b}^U) - \Psi(\tilde{b}^U) = 0. \tag{19}$$

Fixing χ and totally differentiating (19), we obtain

$$\frac{\partial \Phi(\tilde{b}^U)}{\partial \xi} d\xi - \frac{\partial \Psi(\tilde{b}^U)}{\partial \xi} d\xi + \frac{\partial \Phi(\tilde{b}^U)}{\partial \tilde{b}^U} d\tilde{b}^U - \frac{\partial \Psi(\tilde{b}^U)}{\partial \tilde{b}^U} d\tilde{b}^U = 0 \;\Rightarrow\; \frac{d\tilde{b}^U}{d\xi} = \frac{\Phi_\xi(\tilde{b}^U) - \Psi_\xi(\tilde{b}^U)}{-\Phi_{\tilde{b}^U}(\tilde{b}^U) + \Psi_{\tilde{b}^U}(\tilde{b}^U)}, \tag{20}$$

where $\Phi_x(\tilde{b}^U)$ and $\Psi_x(\tilde{b}^U)$ indicate partial differentiations of $\Phi(\tilde{b}^U)$ and $\Psi(\tilde{b}^U)$ with respect to x, respectively. Equation (20) indicates how change of ξ (with fixed χ) affects $\tilde{b}^U$ in the unstable BGP. The sign of the denominator in equation (20) is positive; $-\Phi_{\tilde{b}^U}(\tilde{b}^U) + \Psi_{\tilde{b}^U}(\tilde{b}^U) > 0$. The sign of the numerator in Equation (20) is calculated as

$$\Phi_{\xi}(\tilde{b}^{U}) - \Psi_{\xi}(\tilde{b}^{U}) = A^{1/\alpha}\xi^{1/\alpha-1}\left[\frac{\beta}{1+\beta}(1-\alpha)\frac{(1/\alpha-1)\xi^{-1} - 1/\alpha(1-\chi)}{(1+\alpha\tilde{b}^{U})} - \frac{\chi}{\alpha}\left(\frac{1}{\tilde{b}^{U}}+1\right)\right]. \tag{21}$$

Thus, if equation (21) is positive, an increase in ξ leads to a more sustainable fiscal policy. Equation (21) can be positive when ξ is small.

However, it is difficult for us to obtain information on $\partial\tilde{b}^{U}/\partial\xi$ analytically and hence we use a numerical example of the sign of $\partial\tilde{b}^{U}/\partial\xi$. Accordingly, in the next section, we numerically investigate whether a raising ξ leads to a more sustainable fiscal policy.

2.5 Policy Effects on Welfare

In this subsection, we analytically study the welfare analysis. We show that the current household achieves higher utility if an increase in the public-spending/GDP ratio leads to a more sustainable fiscal policy. This can be proved through a short calculation. A household born at $t-1$ has the indirect utility, V_{t-1}, which is derived as

$$\begin{aligned} V_{t-1} := (1-\beta)\ln\frac{1}{1+\beta} + \beta\ln\beta + (1+\beta)\ln\left[\frac{1-\xi(1-\chi)}{(1+\tilde{b}_{t-1})}(1-\alpha)A^{1/\alpha}\xi^{1/\alpha-1}\right] \\ + (1+\beta)\ln k_{t-1} + \beta\ln\left[1+\frac{1-\xi(1-\chi)}{(1+\alpha\tilde{b}_{t})}\alpha A^{1/\alpha}\xi^{1/\alpha-1}\right] \end{aligned} \tag{22}$$

where k_{t-1}, b_{t-1}, and b_t are predetermined and thus remain constant in the period t. We can make some further calculations and obtain that $\partial V_{t-1}/\partial\xi > 0$ if the increase in the public-spending/GDP ratio, ξ, brings more sustainable fiscal policy. The increase in the public-spending/GDP ratio makes both the productivity and the burden of taxation higher, and the former effect is larger than the latter if it leads to more sustainable fiscal policy.

However, it is difficult to investigate the welfare effect of the households in the future, especially those on the transition path. The reason is that two additional opposite effects arise from an increase in the public-spending/GDP ratio and it is difficult to determine analytically which effect is larger.

One effect is increasing after-tax productivity. It gives higher output growth and positively affects the households' welfare. The other effect is increasing the public-debt/capital ratio. It worsens the welfare through increasing payments of interest and crowding out private investments.

Note that raising the public-spending/GDP ratio may increase the public-debt/capital ratio even if the raise leads to a more sustainable fiscal policy. This implies that the raise can worsen the welfare of near-present generations' households. At (or near) present, the effect of a policy change on output growth is not relative in comparison to the effect on change of the stock of public-debt.

Hence, it is worth analyzing the welfare effect, especially on the transition path. We therefore carry out a numerical exercise to analyze the effect on future generations' welfare in the next section.

3 Numerical Analysis of Policy Effects

In this section, we provide some results of numerical analysis of policy effects on the sustainable fiscal policy and households' welfare. Our results are threefold as follows. First, we illustrate what kinds of fiscal policies are sustainable. Second, we investigate whether raising the public-spending/GDP ratio leads to a more sustainable fiscal policy. Third, we analyze the policy effects on welfare of households of each generation.

3.1 Sustainable Fiscal Policy

We aim to determine what kinds of fiscal policies are sustainable in this subsection. Figure 3 illustrates the range of policy parameters that leads to sustainable fiscal policies with $\alpha = 0.2, \beta = 0.55$ and $A = 12$. These parameter choices follow Bräuninger (2005)[7].

[7] Here, we want to compare the set of policy parameters that leads to fiscal sustainability and adopt the same parameters as those of Bräuninger (2005). Then we do not consider whether or not these parameters are plausible in

[Figure 3 about here.]

In Figure 3, the region below the dotted line represents a sustainable fiscal policy in the economy as considered by Bräuninger (2005). On the other hand, the region below the solid line indicates the sustainable fiscal policy in our model,

We can intuitively interpret the set of parameters that leads to a sustainable fiscal policy as follows. First, we demonstrate why the public-spending/GDP ratio, ξ, must be moderate for achieving fiscal sustainability. In our model, public spending is productive, and the economy-wide production function is $y_t = A^{1/\alpha}\xi^{(1-\alpha)/\alpha}k_t$. Thus, if ξ is very small, the productivity is also very low. This leads to lower economic growth rate, and thus, the higher debt-issuance/public-spending ratio cannot be sustained. Meanwhile, in Bräuninger (2005), public spending is assumed to be wasteful, and the economy-wide production function is $y_t = Ak_t$. Thus, if ξ is small, the government needs few resources to finance the public spending, and can sustain the higher debt-issuance/public-spending ratio, χ [8]. As above, on the left side of Figure 3, the shapes of the graphs differ. On the other hand, if ξ is very high, the 'cost', which comprises a flat-rate income tax and issuance of public debt, increases far more than the increase in marginal productivity and thus worsens fiscal sustainability. These facts correspond to the propositions of Barro (1990), and indicate the inverse-U shaped relationship between the public-spending/GDP ratio and growth rate.

Second, we explain the relationship between public debt-spending ratio, χ, and fiscal sustainability. When the government adopts a higher χ, the government issues so much public debt that

this model. In following section, we again set plausible parameters.

[8]Moreover, if ξ is very small, the productivity in our model is smaller than the one in Bräuninger (2005) by the specification of the economy-wide production functions. This leads to the result that the parameter space of sustainable fiscal policies with productive government spending is not larger than the space without productive government spending.

the payment of interest on public debt grows and private capital is crowded out in the long term. This means that the higher χ the government chooses, the less fiscal sustainability the economy attains.

Figure 3 implies that an increase in ξ may have a positive effect on fiscal sustainability when ξ is relatively small in our model, which is not mentioned in previous studies on fiscal sustainability. The upward sloping graph in Figure 3 indicates that an increase in ξ leads to greater sustainability in fiscal policy, because higher χ worsens fiscal sustainability as shown earlier. We will discuss our findings in detail in the following subsection.

3.2 Policy Effect on Fiscal Sustainability

We numerically analyze policy effects on the maximum sustainable debt/capital ratio in this subsection.

[Figure 4 about here.]

Figure 4 illustrates the relationship between public-spending/GDP ratio, ξ, and the public-debt/capital ratio in an unstable BGP, $\tilde{b}^U$ [9].

The values of α and β are the same values as those in Bräuninger (2005). A is selected as the real interest rate that matches the actual average interest rate of 1-year US treasury bills for 31 years (1973–2003) at around 6.5%. In order to set A, we use $\xi = g/y = 0.203$, which is the average public-spending/GDP ratio[10] from 1973 to 2003. Using the parameters as above, we can set $A = 7.4$ because the interest rate is represented as below in our model,

$$(1+r)^{1/30} = (1 + A\alpha\xi^{1/\alpha - 1})^{1/30} = 1.065. \tag{23}$$

[9] We assume that $\chi = 0.1$. This result does not change under various χ.

[10] The public expenditure is calculated by subtracting the federal payments for individuals from the total government expenditures containing federal, state, and, local governments' expenditures.

We set the debt-issuance/public-spending ratio, χ, as $\chi = 0.1$ in Figure 4. However, this result does not change under various χ.

We can observe the non-monotonic relationship between the public-spending/GDP ratio, ξ, and the maximum sustainable debt/capital ratio, $\tilde{b}^U$. Previous studies on fiscal sustainability, for example, Chalk (2000), Bräuninger (2005), Yakita (2008), and so on, have not reported the inverse-U shaped relationship.

We now consider the reason why the non-monotonic relationship arises. Assuming that the public-spending/GDP ratio is small, we observe that an increase in the public-spending/GDP ratio drastically raises marginal productivity. This effect surpasses the other negative effects on fiscal sustainability, such as an increase in tax rate, more issuance of public debt, and crowding out of private capital, which contribute to the sustainability of the fiscal policy rules. As a result, when ξ is very small, raising ξ leads to a more sustainable fiscal policy. Inversely, if the public-spending/GDP ratio is relatively large, when the government raises ξ, the negative effects surpass the effect of the increase in marginal productivity. Thus, raising ξ leads to a less sustainable fiscal policy when ξ is relatively large.

3.3 Fiscal Sustainability versus Growth-Maximizing Policy

Here, we consider the following question: does the growth-maximizing fiscal policy always make public debt sustainable? Our answer is No. This is shown by a numerical exercise.

[Figure 5 about here.]

Figure 5 illustrates the relationship between public-spending/GDP ratio, output growth rate, and the public-debt/capital ratio in the unstable BGP, $\tilde{b}^U$[11].

[11]We assume that $\chi = 0.1$. This result does not change under various χ.

In Figure 5, the peak of the graph of sustainable initial debt/capital ratio is located left of the peak of the graph of output growth rate. Figure 5 illustrates that the growth-maximizing public-spending/GDP ratio does not always make public debt sustainable. This implies that the growth-maximizing fiscal policy is not always more sustainable than the other fiscal policies. For example, suppose that $\chi = 0.1$ [12] and the initial public-debt/capital is 9, $\tilde{b}_0 = 9$. If the government chooses the growth-maximizing public-spending/GDP ratio, $\xi = 0.555$, the government cannot sustain public debt because the upper limit of the initial public-debt/capital ratio for which the debt/capital ratio converges, $\tilde{b}^U$, is 5.301. However, if the government changes ξ to $\xi = 0.395$, the government can sustain public debt because $\tilde{b}_0 < \tilde{b}^U = 9.499$.

Barro (1990) assumes that government spending is productive and obtains the result that there exists a hump-shaped relationship between public spending and economic growth. Meanwhile, in our model, we also assume productive government expenditure and show that there exists an inverse-U shaped relationship between public spending and the largest sustainable public debt. These results may appear similar, but we identify a new implication in this paper. We find that the growth-maximizing fiscal policy does not lead to the largest sustainable (initial) public debt in this model. In other words, the fiscal policy that gives the largest sustainable public debt differs from the growth-maximizing policy.

The reason why the growth-maximizing public-spending/GDP ratio gives the largest sustainable public debt is that an increase in public spending raises not only output growth but also issuance of public debt. Issuance of public debt leads to an increase in the growth rate of public debt, a falling output growth rate through crowding out of private capital, and an increase in interest payment on public debt.

[12] Thus, Figure 5 is held.

3.4 Welfare Analysis

Although raising the public-spending/GDP ratio results in greater fiscal sustainability and higher growth rate if the ratio is relatively small, this does not necessarily imply that such a policy change is Pareto improving. Strictly speaking, the increase in the public-spending/GDP ratio makes the welfare of the households in future generations higher or lower, while it necessarily increases the welfare of the current households. In this subsection, we analyze how the policy change in the public-spending/GDP ratio affects the welfare of households.

In the numerical analysis, we investigate how present and future generations' welfare changes when the public-spending/GDP ratio rises from the smallest value guaranteeing fiscal sustainability. The procedure of the exercise is as follows:

1. The parameters are set.
   - The key parameters are set as in the previous section[13]: $A = 7.4$, $\alpha = 0.2$, and $\beta = 0.55$.
   - The debt-issuance/public-spending ratio, χ, is fixed to 0.1, 0.2, 0.3, or 0.38[14].
   - The initial state variables, B_0 and K_0, are set as $B_0 = 0.9$ and $K_0 = 1.0$[15].
2. We fix the debt-issuance/public-spending ratio. The public-spending/GDP ratio is set to the smallest value among sustainable fiscal policy parameters, and we calculate the welfare of all generations.

[13]The parameters are set to match those of the US data in 1973–2003. These parameters are the same values as used in section 3. See section 3.2, p.18.

[14]If the debt-issuance/public-spending ratio, χ, is over 0.385, the economy has no steady state, that is, no fiscal policies are sustainable for any level of initial public-debt/capital ratio and public-spending/GDP ratio (ξ).

[15]The reason we set the initial state variables as listed is that we focus on the dynamic paths for the public-debt/capital ratio to converge in the welfare analysis. The value would not correspond to the actual value. However, we consider that the initial stock variables do not affect the result of this numerical exercise and thus this setting of the initial public-debt/capital ratio is not a problem.

3. The public-spending/GDP ratio is increased, as long as the increase gives a more sustainable fiscal policy, and we compare the welfare of all generations[16] before the policy changes with those after.

From the numerical exercise, we conclude that, with the fixed debt-issuance/public-spending ratio, the welfare of all generations is improved by the change of the public-spending/GDP ratio which leads to a more sustainable fiscal policy[17]. It is important that future generations' welfare is improved by the policy change, while it is natural that the policy change must also improve the present generation's welfare. In order to intuitively understand the result, the effects of the increase in the public-spending/GDP ratio are summarized again as follows:

1. increase in the productivity and the capital growth rate, (short-term and long-term, positive effect)

2. increase in income tax rate, (short-term, negative effect)

3. increase in the tax burden and enlargement of the crowding-out effect on private investment by accumulation of public debt. (long-term, negative effect)

This result implies that the negative long-term effect (Effect 3) is relatively small compared to the other effects (Effects 1 and 2). In other words, when raising the public-spending/GDP ratio gives a more sustainable fiscal policy, the growth-enhancing effect is much larger than the negative effect of accumulation of public debt and improves the welfare of all generations.

[16]We calculate the welfare of the households from present to 14 periods after, which corresponds to about 350-500 years after. After 14 periods, the growth rate of the debt-capital ratio is almost zero and we can consider that the economy reaches the steady state.

[17]The numerical result is illustrated in Appendix B.

4 Conclusion

We have constructed an overlapping generations model in which the government engages in productive public spending, and conforms to the constant public-spending/GDP and public debt-spending rules. We then numerically analyze the policy effects on fiscal sustainability, growth rate, and welfare of households.

We obtain the two main results. First, we obtain the inverse-U shaped relationship between the public-spending/GDP ratio and fiscal sustainability: if the public-spending/GDP ratio is small (large), raising the ratio leads to a more (less) sustainable fiscal policy. Moreover, the output growth-maximizing fiscal policy does not always make public debt sustainable. This implies that the growth-maximizing fiscal policy may involve too large a public-spending/GDP ratio to sustain public debt.

Second, when the public-spending/GDP ratio is small, an increase in the spending/GDP ratio is Pareto improving. In other words, if an increase in the public-spending/GDP ratio results in a more sustainable fiscal policy, then such a policy change improves the welfare of all generations. Note that, when an increase in the public-spending/GDP ratio leads to a less sustainable policy, such a policy change is not Pareto improving in general.

Here, we present Figure 6. Figure 6 is the graphical summary of our results.

[Figure 6 about here.]

The non-monotonic relationship between the size of the public-spending/GDP ratio and fiscal sustainability implies that we have to take care of not only the initial stock of public debt but also the size of the public-spending/GDP ratio parameter in order to attain fiscal sustainability through policy changes. When ξ is relatively small, "fiscal reforms" involving cuts in public spending may negatively affect fiscal sustainability, which was not examined in previous studies on fiscal sustainability, such as Bräuninger (2005) and Yakita (2008). Meanwhile, an increase in public-spending, which gives higher output growth, may not become a more sustainable fiscal policy. This implies that it is possible for a "higher economic growth" policy to lead to less fiscal sustainability.

Appendix

A Derivation of Equation (16)

In this section, we give an explanation of how to obtain equation (16).

First, we compute $(1-\tau_t)$. Total tax revenue, T_t, is

$$T_t = \tau_t\left(r_t s_{t-1} + w_t\right) = \tau_t(y_t + r_t b_t) \tag{24}$$

from Equations (8), (9), and (11). Then, the government's budget constraint, (4), and policy rules, (5) and (6), leads to

$$\xi\chi y_t = r_t b_t + \xi y_t - \tau_t(y_t + r_t b_t). \tag{25}$$

We solve Equation (25) for τ_t and obtain

$$1-\tau_t = \frac{1+\xi\chi-\xi}{1+\alpha\tilde{b}_t}. \tag{26}$$

Second, we calculate k_{t+1}/k_t and b_{t+1}/b_t. k_{t+1}/k_t is derived by substitution of Equations (7), (26) and (6) to (11) as

$$\frac{k_{t+1}}{k_t} = \frac{\beta}{1+\beta}\frac{1+\xi\chi-\xi}{1+\alpha b_t/k_t}A^{1/\alpha}\xi^{1/\alpha-1}(1-\alpha) - \frac{b_t}{k_t} - \chi A^{1/\alpha}\xi^{1/\alpha}. \tag{27}$$

Further, we obtain b_{t+1}/b_t from Equation (6),

$$\frac{b_{t+1}}{b_t} = 1 + \chi\left(\frac{b_t}{k_t}\right)^{-1} A^{1/\alpha}\xi^{1/\alpha} \tag{28}$$

Finally, combining (27) and (28), we get equation (16),

$$\frac{b_{t+1}/b_t}{k_{t+1}/k_t} = \frac{\tilde{b}_{t+1}}{\tilde{b}_t} = \frac{1 + \chi \tilde{b}_t^{-1} A^{1/\alpha} \xi^{1/\alpha}}{\frac{\beta}{1+\beta} \frac{1+\xi\chi-\xi}{1+\alpha\tilde{b}_t} A^{1/\alpha} \xi^{1/\alpha - 1}(1-\alpha) - \tilde{b}_t - \chi A^{1/\alpha} \xi^{1/\alpha}}. \tag{29}$$

B The Result of The Numerical Exercise on Welfare Analysis

We illustrate the results of the numerical analysis on each generations' welfare by plotting the welfare of households. In the following figures, the horizontal axis represents the number of generations (from present to 14 periods after), and the vertical axis represents each generations' welfare.

[Figure 7 about here.]

[Figure 8 about here.]

[Figure 9 about here.]

[Figure 10 about here.]

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Figure 1: *Existence of BGPs*

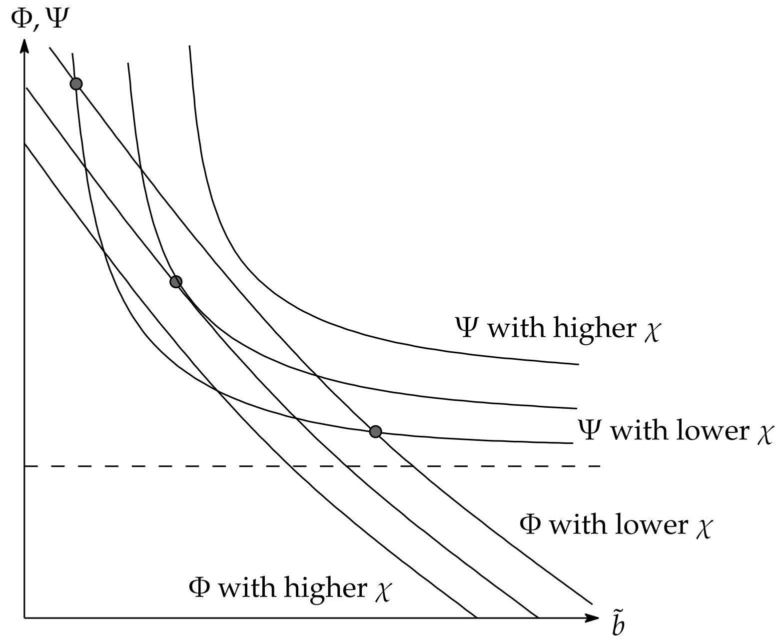


Figure 2: *The Stability of BGP When There Exists Two BGPs*

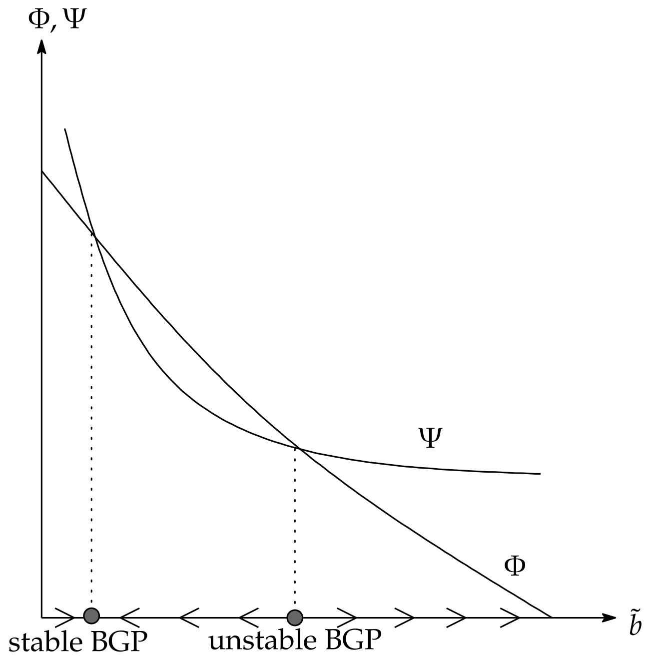
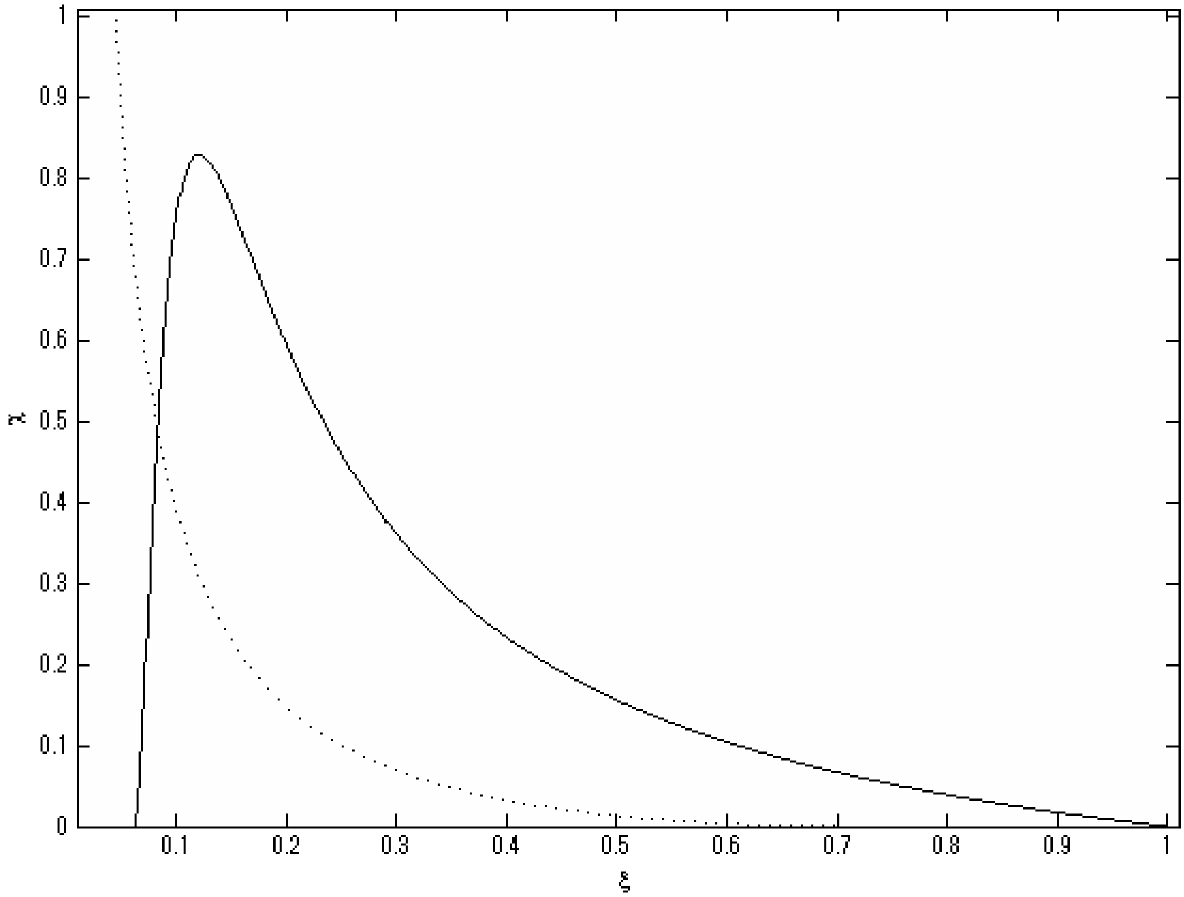
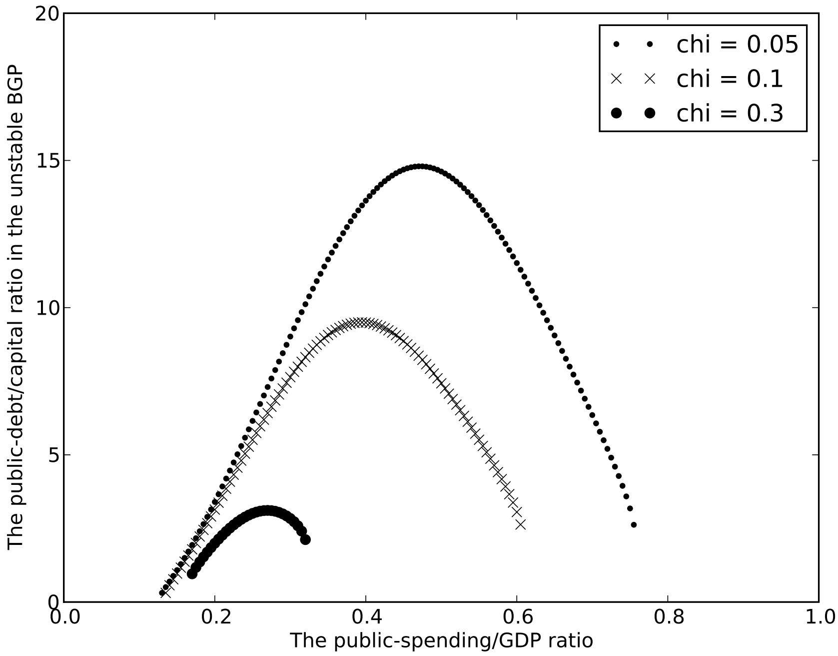


Figure 3: *The Range of Sustainable Fiscal Policy Parameters*



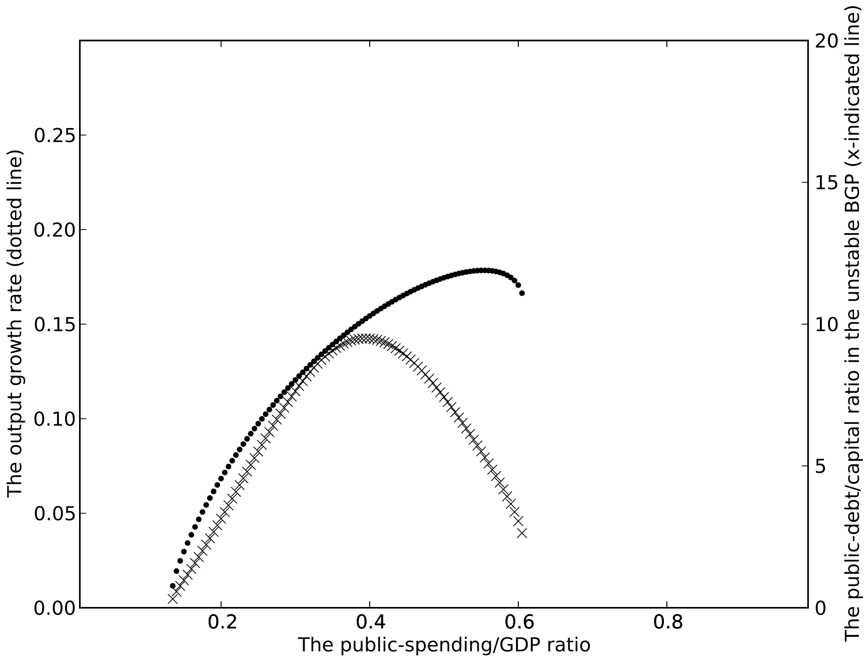
Note: The region below the line indicates the area of sustainable fiscal policy, (ξ, χ). [Solid line: productive spending spending (this paper); dotted line: wasteful public spending (Bräuninger, 2005)]

Figure 4: *The Relation Between $\tilde{b}$ in the Unstable BGP and ξ*



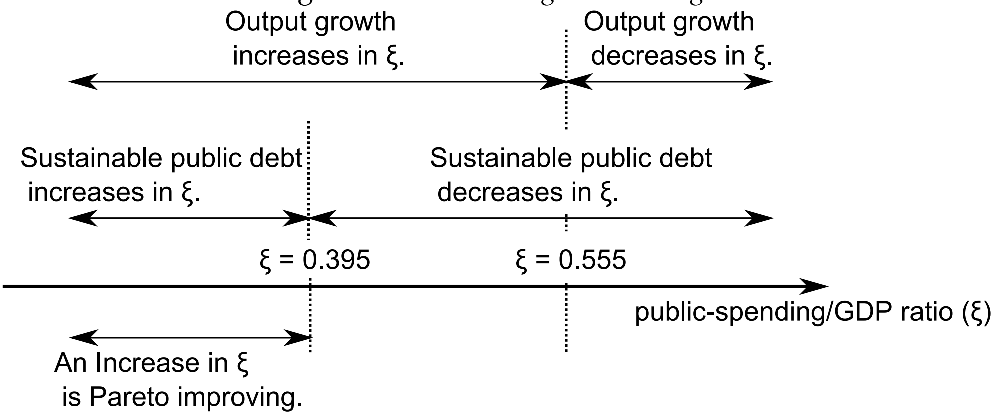
Note: We set the value of χ, the issuance-debt/public-spending ratio, as $\chi = 0.1$. The horizontal axis corresponds to the value of the public-spending/GDP ratio. The vertical axis corresponds to the public-debt/capital ratio in the unstable BGP (the measurement of fiscal sustainability).

Figure 5: *The Relationship Between the Public-spending/GDP Ratio, Output Growth rate, and the Public-debt/capital Ratio in the Unstable BGP*



Note: The horizontal axis corresponds to the public-spending/GDP ratio, ξ. The first vertical axis (left-side axis) corresponds to the output growth rate. The dotted line illustrates the relationship between the public-spending/GDP ratio and the output growth rate. The second vertical axis (right-side axis) corresponds to the public-debt/capital ratio in the unstable BGP. The 'x'-indicated line illustrates the relationship between the public-spending/GDP ratio and the public-debt/capital ratio in the unstable BGP.

Figure 6: *Summarizing our Findings*



Note: Figure 6 illustrates the relationship between the public-spending/GDP ratio, ξ, fiscal sustainability, and output growth rate given the debt-issuance/public-spending ratio, χ. Moreover, Figure 6 states that an increase in ξ is Pareto improving if ξ is small. The numerical values are calibrated with $\chi = 0.1$.

Figure 7: *Welfare Analysis (The case of χ = 0.10)*

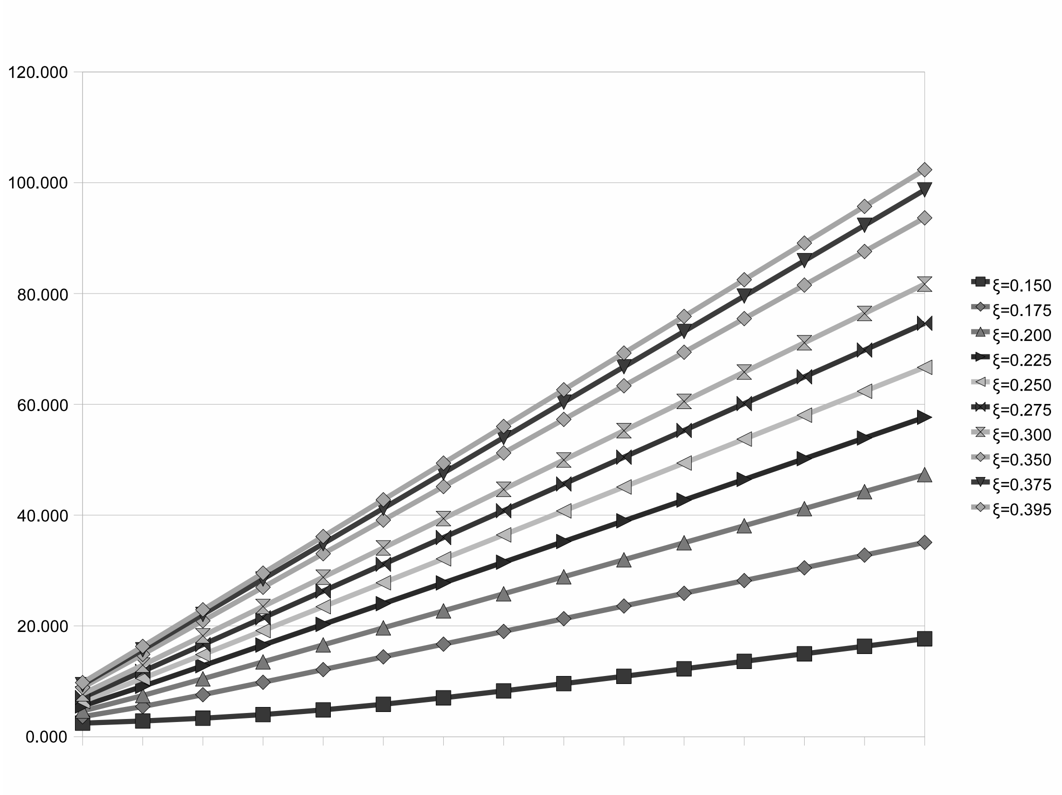


Figure 8: *Welfare Analysis (The case of χ = 0.20)*

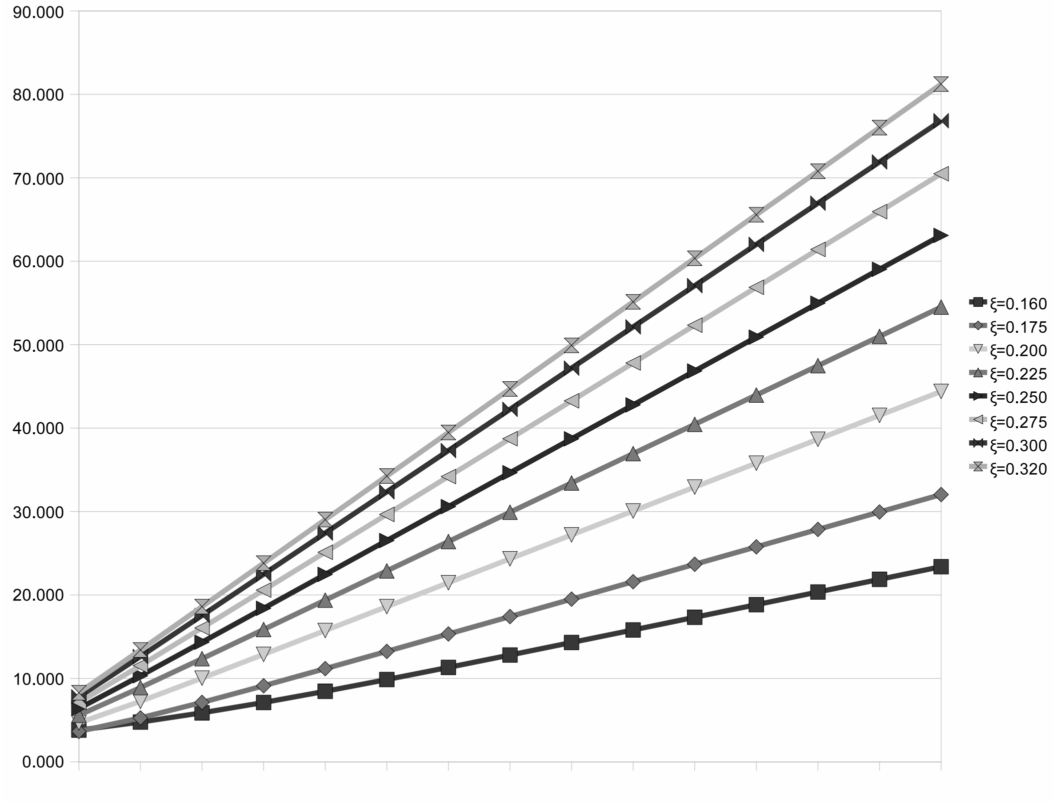


Figure 9: *Welfare Analysis (The case of χ = 0.30)*

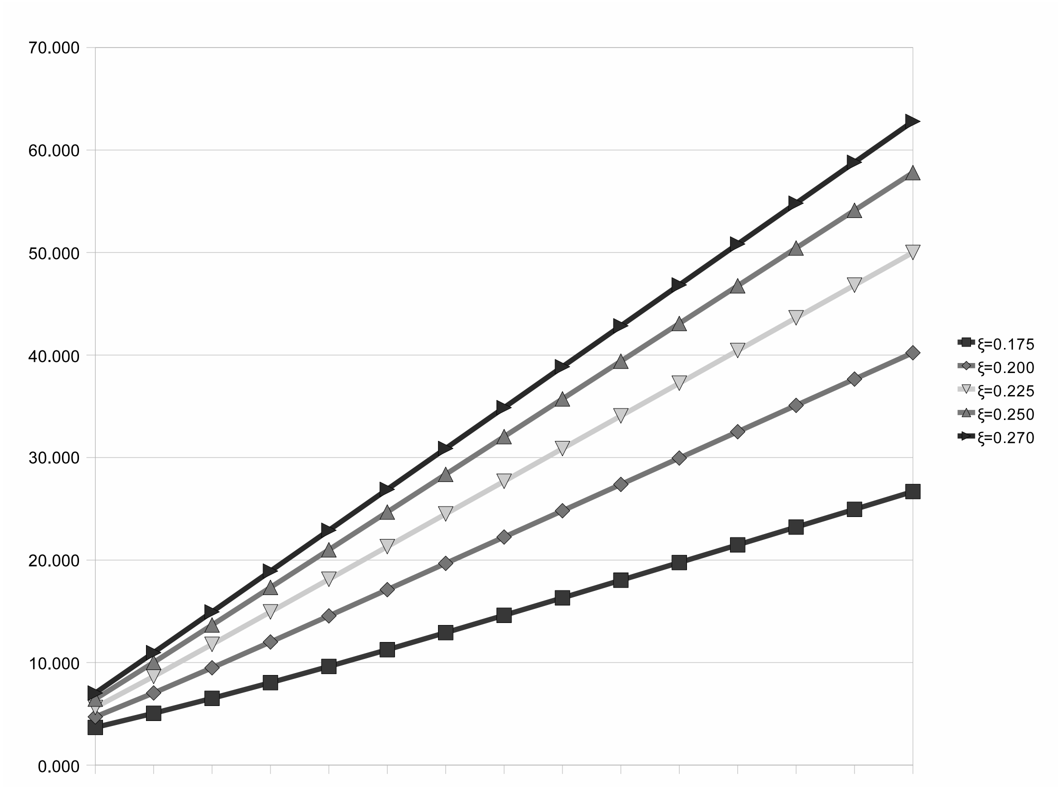


Figure 10: *Welfare Analysis (The case of* $\chi = 0.38$*)*